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GUIDE FOR EVALUATION OF UNCERTAINTY IN CALIBRATION

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Symbols and Abbreviations Used

$\sum$	:	Sigma, which means Summation
α	:	Coefficient of thermal expansion (…µ per ^{o}C per meter, OR …µ" per ^{o}F per inch)
σ	:	Standard deviation
k	:	Coverage factor
DOF	:	Degrees of freedom
n	:	Number of repeated readings
% Rh	:	Percentage relative humidity
t	:	Temperature
t_s	:	Temperature at start of exercise
t_e	:	Temperature at end of exercise
UUC	:	Unit under calibration
U_A, U_{A1}, U_{A2}	:	Type 'A' uncertainties
U_B, U_{B1}, U_{B2}	:	Type 'B' uncertainties
U_c	:	Combined uncertainty
U_{95}	:	Expanded Uncertainty at 95% confidence level
X_1, X_2, X_3	:	Individual readings
$\overline{X}$	:	Mean (average) of readings

METHODOLOGY FOR EVALUATION OF UNCERTAINTIES

1. Instrument details: (Unit under calibration)
   - Name:
   - Identification number:
   - Range:
   - Resolution:

2. Calibration procedure:
   - Procedure number:
   - Last revised on:
   - Reference for procedure:

3. Date / Location of calibration:

4. Reference standards used:

   (a) Name/Grade/Class:

   Traceability / Cal Cert Number:

   Uncertainty of Calibration:

   Deviation from nominal value:

   (b) Name/Grade/Class:

   Traceability/Cal Cert Number:

   Uncertainty of Calibration:

   Deviation from nominal value:

   [Use (c), (d), etc., if additional reference standards are used]

5. Standard reference temperature/humidity: 68°F (20°C) / 50±5% Rh (for dimensional)

6. Actual environmental conditions:
   a) Temperature:
      - At start (t_s): __°C
      - At end (t_e): __°C
      - Average: $(t_s + t_e)/2$: __°C
      - Thermometer resolution: __°C

   b) Humidity % Rh

7. Repeatability Measurements (Type 'A' Evaluation):

Serial No.	Readings(X_i)	Mean ($\bar{x}$)	$(X_i - \bar{x})^2$	$\sum(X_i - \bar{x})^2$	Standard deviation
1	X_1	$\bar{x}$ is equal to $\frac{\sum X_n}{n}$	$(X_1 - \bar{x})^2$	$\sum (X_i - \bar{x})^2$	$\sqrt{\frac{\sum(X_i - \bar{x})^2}{(n-1)}}$
2	X_2		$(X_2 - \bar{x})^2$		
3	X_3		$(X_3 - \bar{x})^2$		
4	X_4		$(X_4 - \bar{x})^2$		
5	X_5		$(X_5 - \bar{x})^2$		
6	X_6		$(X_6 - \bar{x})^2$		
7	X_7		$(X_7 - \bar{x})^2$		
8	X_8		$(X_8 - \bar{x})^2$		
9	X_9		$(X_9 - \bar{x})^2$		
10	X_{10}		$(X_{10} - \bar{x})^2$		

σ = Standard deviation of the above 10 repeated readings = $\sqrt{\frac{\sum(X_i - \bar{x})^2}{(n-1)}}$

To arrive at the random uncertainty of the population, the standard deviation of the mean has to be evaluated. This is obtained by dividing the standard deviation of the sample by √n.

Standard deviation of the mean = σ / √n

This is the type 'A' uncertainty or

Random Uncertainty $U_{A1} = \sigma / \sqrt{n}$

If there is more than one contributor for type 'A' uncertainties, repeat the above process for each one of them. Term them as U_{A2}, U_{A3}, U_{A4}, etc.

Then, **Random Uncertainty,** $U_A = \sqrt{U^2_{A1} + U^2_{A3} + \ldots + U^2_{An}}$

8. Type ‘B’ Evaluation:

While considering the various contributory factors towards type B uncertainties, one has to analyze the calibration process itself, if necessary by breaking it down into a number of small processes and studying the influences that are likely to affect the process. Generally, type B factors may include but are not limited to: the reference standard, the UUC itself, other supporting instruments involved in the calibration process, personnel carrying out the calibration, and environmental factors such as temperature, pressure, humidity, air density, local “g” value, etc. Any possible interaction between the above factors may also need to be taken into account.

Generally, the following factors should be included in any type ‘B’ evaluation.

U_{B1}: Uncertainty contribution due to deviation of the reference standard from its nominal value. (Assume rectangular distribution and coverage factor of $\sqrt{3}$.)

U_{B2}: Uncertainty due to the uncertainty of calibration of reference standard (value obtained from the calibration certificate of the reference standard from a higher level laboratory or NMI. Coverage factor is also obtained from the certificate and is usually k = 2, for normal distribution).

U_{B3}: Uncertainty contribution due to resolution of UUC (take half resolution & $k=\sqrt{3}$).

U_{B4}: Uncertainty contribution due to resolution of the temperature measuring device (take half resolution and distribution as rectangular – use sensitivity coefficient to account for input/output mismatch).

U_{B5}: Uncertainty contribution due to uncertainty of the temperature measuring device (this value & the coverage factor, k, are obtained from the calibration certificate).

U_{B6}, U_{B7}, U_{B8}, U_{B10}, etc.

(These contributory factors are considered depending on the actual calibration being carried out and keeping in mind that there is no limit to the number of factors to be considered.)

After evaluating all the contributory factors, the combined uncertainty has to be evaluated, the degrees of freedom must be calculated and the final expanded uncertainty determined.

9. Uncertainty Budget:

Serial No.	Source of Uncertainty & Value	Probability Distribution Factor	Standard Uncertainty	Sensitivity Coefficient*	Uncertainty Contribution	Degrees of Freedom
1	Type A	Normal – 2		1	U_{A1}	(n-1)
2	Type B			1	U_{B1}	∞
3	Type B			1	U_{B2}	∞
4	Type B			1	U_{B3}	∞
5	Type B			1	U_{B4}	∞
6	Type B			1	U_{B5}	∞
7	Type B			1	U_{B6}	∞
8	Type B etc			1	U_{B7} etc	∞

[*The sensitivity coefficient depends on the units for input and output quantities (for example, temperature input in °C and linear measure output in microns) into the measuring system and has to be worked out depending on the particular calibration being carried out.]

Now, Combined Uncertainty,

$$\mathbf{U_c = \sqrt{U^2_A + U^2_{B1} + U^2_{B2} + \ldots + U^2_{Bn}}}$$

Effective Degrees of Freedom $V_{eff} = \dfrac{U^4_C}{\dfrac{U^4_A}{(n-1)} + \dfrac{U^4_{B1}}{DOF} + \dfrac{U^4_{B2}}{DOF} + \cdots}$

This V_{eff} should be a large number.

10. Refer to the “Student T” table to obtain the “k” factor for the number of repeatability readings taken and the intended confidence level (usually 95% for calibration laboratories).

11. Expanded uncertainty at 95% confidence level is:

$$\mathbf{U_{95} = U_C \times \text{‘k’ Factor}}$$

Type ‘B’ Factors to be Considered for Various Disciplines

Following are the factors that generally need to be considered for type ‘B’ evaluation of uncertainties. However, it may be worthwhile remembering that this list is not exhaustive and also that there is no limit to the consideration of contributory factors. From the list, choose only the factors that are relevant for the particular calibration exercise that is being carried out.

I. Dimensional (consider whatever factors are relevant for the exercise being done)

1) Deviation of reference standard from its nominal value ($k=\sqrt{3}$)

2) Uncertainty of calibration of reference standards (from certificate, k=2)

3) Resolution of UUC (consider half resolution & $k=\sqrt{3}$)

4) Resolution of thermometer (consider half resolution & $k=\sqrt{3}$)

5) Uncertainty of calibration of thermometer (value from certificate and k=2)

6) Difference between standard temperature and average temperature of calibration ($k=\sqrt{3}$)

7) Nonuniform expansion because of temperature difference between reference standard and UUC. (Consider this to be 20% of (6) and rectangular distribution - $k=\sqrt{3}$)

8) Uncertainty of thermal coefficient of expansion of UUC
(Consider this to be 10% of the value and rectangular distribution. $K=\sqrt{3}$.)

9) Uncertainty of thermal coefficient of expansion of reference standard
(Consider this to be 10% of the value and rectangular distribution. If both UUC and reference standard are of the same material, combine (8) and (9) and take 20% - $k=\sqrt{3}$.)

10) Flatness of surface plate (10-20% of value), if a surface plate is used in the calibration ($k=\sqrt{3}$).

11) Uncertainty of flatness of surface plate, if a surface plate is used in the calibration ($k=\sqrt{3}$).

12) Flatness of anvils (e.g., a micrometer & $k=\sqrt{3}$)

13) Parallelism of anvils (e.g., a micrometer & $k=\sqrt{3}$)

14) Parallelism of jaws (e.g., a caliper & $k=\sqrt{3}$), etc.

II. Hardness (Indirect calibration)

(1) Uncertainty of calibration of reference hardness block.

(2) Resolution of hardness tester (converted to hardness value using sensitivity coefficient).

III. Force

(1) Uncertainty of calibration of the proving device

(2) Resolution of force indication

(3) Effect of temperature on proving device

(4) Effect of drift of proving device if no data is available, assume a value equal to (1) i.e., uncertainty of the proving device

(5) Linear interpolation between calibration points (If polynomial curve fit is used, this is negligible.)

IV. Mass

(1) Uncertainty of calibration of reference weight

(2) Resolution of balance

(3) Air buoyancy

(4) Eccentricity effects

(5) Drift

(6) Hysteresis effects.

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UNCERTAINTY EVALUATION FORMAT

1. Name of the laboratory:
2. Unit under calibration: Name:
   Range: Resolution:
3. Standards and equipment used:

Serial No.	Ref.Std / Equipment	Range/ Resol.	U_{95}	Deviation	Traceability
1					
2					
3					
4					

4. Temperature and humidity:

Start	Middle	End	Average	Humidity

5. Type 'A' evaluation:

Serial No.	Readings	Mean (X bar)	(X_i - X bar)	(X_i -X bar) ^{2}	Std. Dev (σ)
1		$\text{X bar} = \frac{\Sigma X_n}{n}$			$\sigma = \sqrt{\frac{\sum(X_i - X\ \text{bar})^2}{(n-1)}}$
2					
3					
4					
5					
6					
7					
8					
9					
10					

U_a i.e., Random Uncertainty is (σ / √n):
Degrees of freedom: (n-1) =

6. <u>Type ‘B’ evaluation</u>:

Sl No	Source	Value	Distribution & ‘k’ factor	Calculations	Std.Unc
1					
2					
3					
4					
5					
6					
7					
8					

7. Uncertainty budget:

Source of Uncertainty	Estimate	Probability Distribution	Standard Uncertainty	Sensitivity Coefficient	Uncertainty Contribution	Degrees of Freedom

Effective Degrees of freedom, V_{eff},

$$= \frac{U_c^4}{U_A^4 / (n-1) + U_{B1}^4 / DOF + U_{B2}^4 / DOF + U_{B3}^4 / DOF + \ldots\ldots + U_{Bn}^4 / DOF^{*}}$$

Coverage factor for 95% confidence level =	
Expanded uncertainty at 95% confidence level = U_c X coverage factor =	

* DOF: Degrees of Freedom

Calibrated by

Approved by

Assessor's comments and signature: